

**FINITE DIFFERENCE ANALYSIS OF INTEGRATED OPTICAL CHANNEL  
WAVEGUIDES WITH ARBITRARILY GRADED INDEX PROFILE**

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**ABSTRACT**

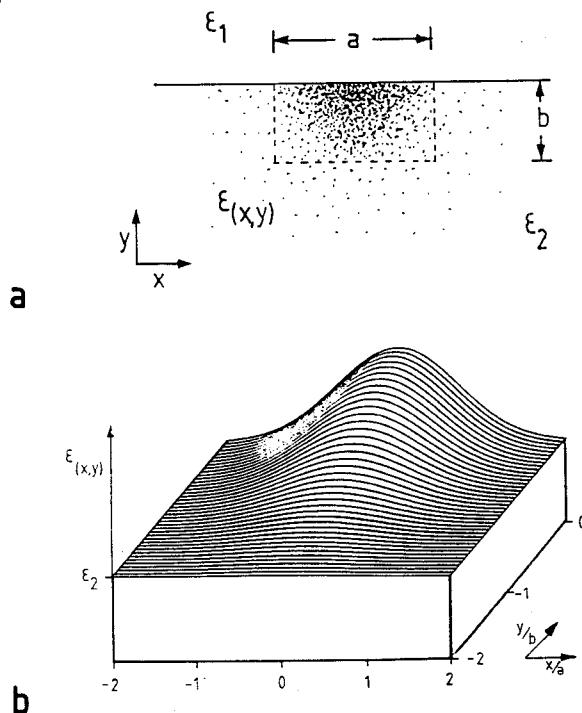
A new finite-difference formulation is described for analyzing diffused dielectric channel waveguides with arbitrarily varying two-dimensional index profiles and arbitrary index difference levels. The method allows the calculation of the complete set of hybrid modes, without the nonphysical, spurious solutions. Hybrid mode dispersion curves of integrated optical channel waveguides with graded index profiles of practical interest are presented.

**INTRODUCTION**

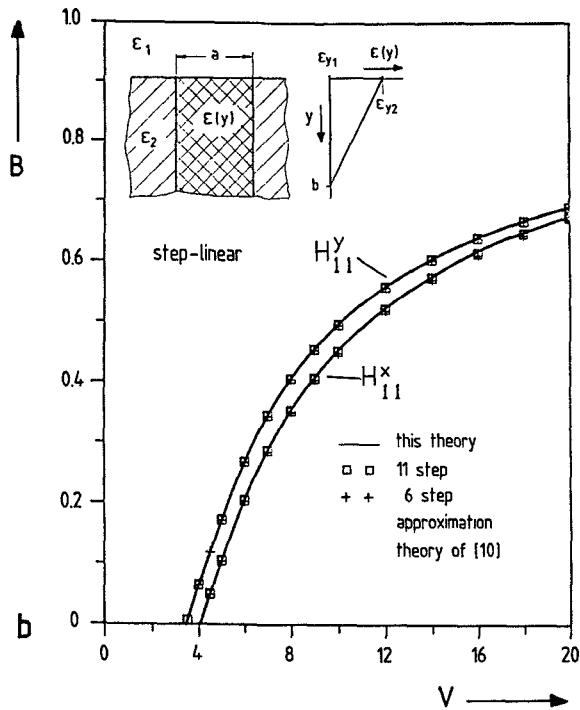
Dielectric channel waveguides have found widespread application in integrated optical devices for the fabrication of phase shifters, modulators, switches, wavelength filters, and couplers [1] - [7]. As such waveguides used in practice are increasingly formed by diffusion techniques [4] - [7], the geometry can be quite complicated, and the channels with graded index profiles in the transverse directions (typically Gaussian and exponential functions) are often not amenable to analytical treatment. The purpose of this paper is to present a new, simple, versatile finite-difference solution for analyzing the hybrid mode propagation on inhomogeneous channel waveguides (Fig. 1) with continuously varying two-dimensional index profiles and arbitrary index difference levels.

Channel waveguides with step index channels have already been analyzed in [1] - [4], [9], [10]. A finite-element solution of one-dimensional diffused channel waveguides has been presented in [4]. However, for channel waveguides with arbitrarily varying two-dimensional index profiles, only approximate techniques have been hitherto applied, such as the Fourier-transform marching analysis for weakly guiding structures [8], and the variational finite-difference method approximation for small index differences [5]. These methods are not appropriate for arbitrary index profile levels. Moreover, the quasi-TE mode solution of [5] ignores the physical reality of hybrid

modes on such waveguide structures, as well as the coupling effects [9] between them. The finite-difference method presented avoids these shortcomings, provides the advantage to yield a reliable, general and flexible computer analysis to allow dominant and higher order hybrid mode solutions of all desired practical cases, and is free from the troublesome problem of nonphysical or "spurious" modes [4]. A graded mesh permits the numerically effective investigation of structures with realistic index profiles by making the mesh finer in regions of particular interest. Related coupled structures are implicated by suitable electric or magnetic wall symmetry. Numerical results compared with available data from other methods verify the theory given.



**Fig. 1: Channel waveguide with arbitrarily varying index profile  $\epsilon(x,y)$**   
a) Diffused channel waveguide with the aspect ratio  $a/b$ . b) Plot of a Gaussian-Gaussian profile.

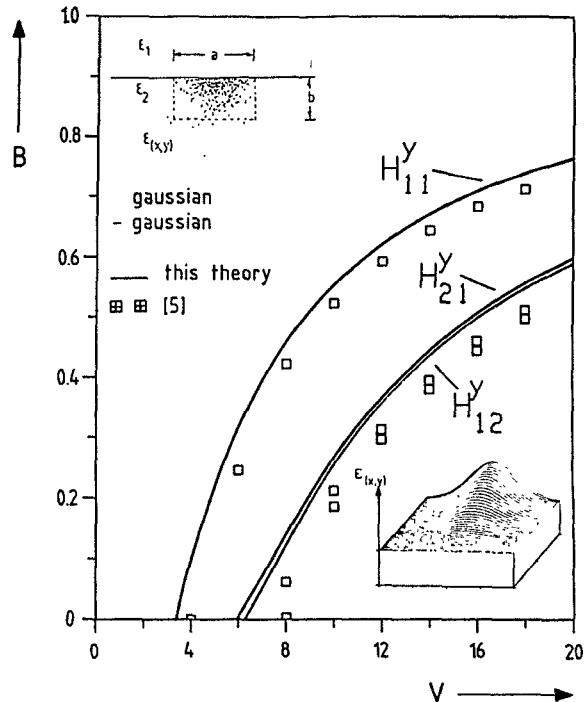


**Fig. 2b:** Comparison with a staircase approximation. Channel waveguide with step profile in x-, and linearly varying profile in y-direction.  $a/b=1$ ,  $\epsilon_1=\epsilon_0$ ,  $\epsilon_2=2\epsilon_0=\epsilon_{y1}$ ,  $\epsilon_{y2}=4\epsilon_0=\epsilon_{\max}$ .

higher order modes  $H_{22}^Y$ ,  $H_{41}^Y$ , only moderate agreement with the finite-element solution ( $14 \times 14$  mesh divisions) of [4] is obtained, but like there, the additional  $H_{41}^Y$  mode is perceived.

For a channel waveguide, where a step index profile in x-direction and a linear profile in y-direction has been assumed, Fig. 2b compares the results of this theory with a staircase approximation of eleven, and six steps, respectively, by using the finite-difference method for rectangular layered structures [10]. Good agreement may be stated.

Fig. 3 presents the normalized propagation characteristics of a practical diffused channel waveguide with a Gaussian-Gaussian profile. Such profiles are realistic in the case of titanium diffusion in  $\text{LiNbO}_3$  and  $\text{Ag}^+ \text{Na}^+$  ion-exchanged glass waveguides [5]. Because of the TE-mode approximation in [5], only poor agreement between our exact theory and the results of [5] may be stated (Fig. 3). Fig. 4 demonstrates the existence of hybrid  $H_{mn}^X$  modes, which, although in close proximity, may be

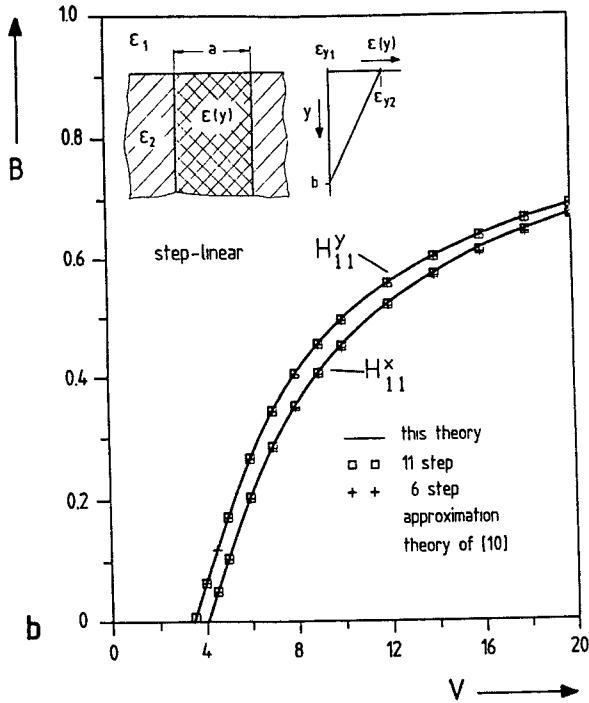


**Fig. 3:** Normalized propagation characteristic B versus normalized frequency V. Practical diffused channel waveguide with a Gaussian-Gaussian index profile, comparison with [5]. Channel waveguide according to [5].

clearly distinguished from the  $H_{mn}^Y$  propagation curves.

Since no spurious modes occur, the finite-difference method described is also well appropriate for analyzing more complicated phenomena at channel waveguides with arbitrary index distribution, such as hybrid-mode crossing effects, which have been observed at step index channel waveguides as well [2], [10]. For a Gaussian-exponential index profile chosen for example, Fig. 5a shows that the dispersion curves of the  $H_{21}^Y$ , and  $H_{31}^Y$  modes, cross those of the  $H_{12}^Y$ , and  $H_{22}^Y$  modes, respectively. This effect is still increased for an exponential-Gaussian index profile (Fig. 5b) concerning the  $H_{12}^Y$ , and  $H_{13}^Y$  modes, which concentrate in the region of higher permittivity with higher velocity than the  $H_{21}^Y$ , and  $H_{31}^Y$  modes, respectively.

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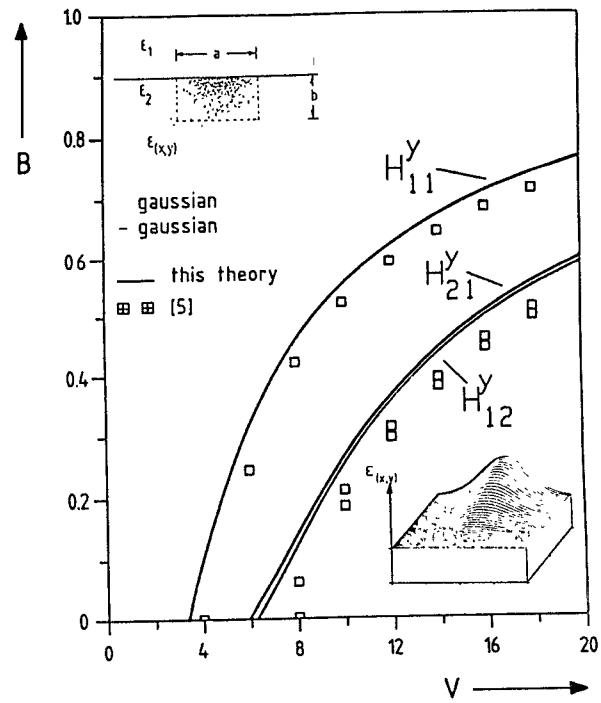


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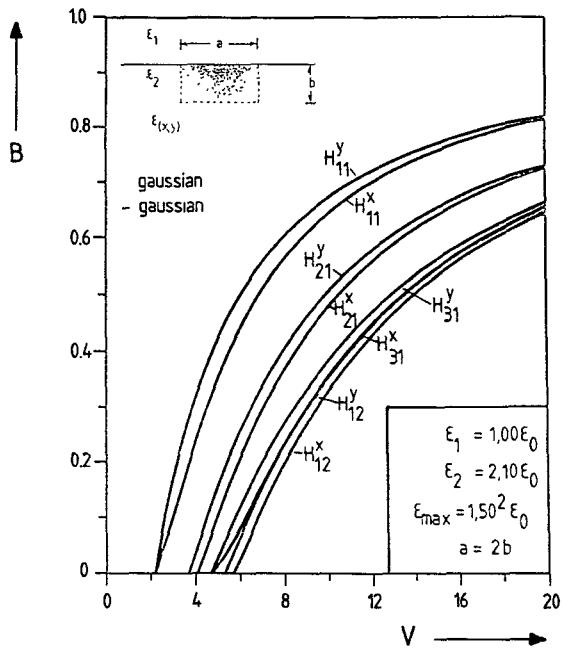


Fig. 4: Normalized propagation characteristic  $B$  versus normalized frequency  $V$ . Diffused channel waveguide with a Gaussian-Gaussian index profile.

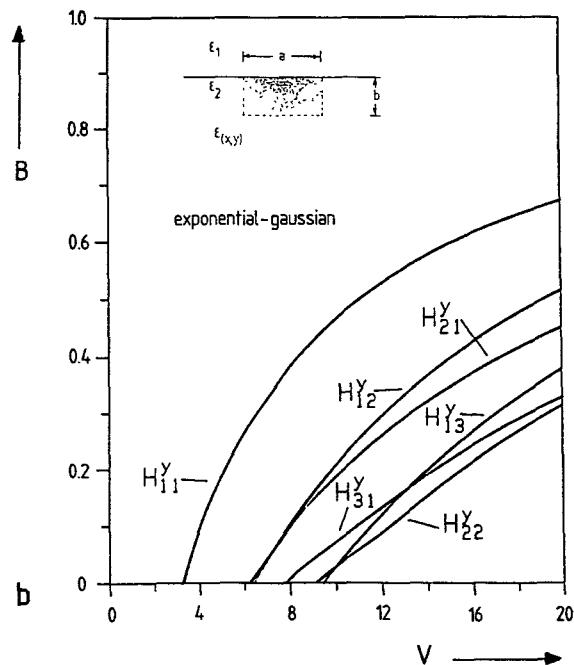


Fig. 5b: Diffused channel waveguide with an exponential-Gaussian index profile. Data same as a).

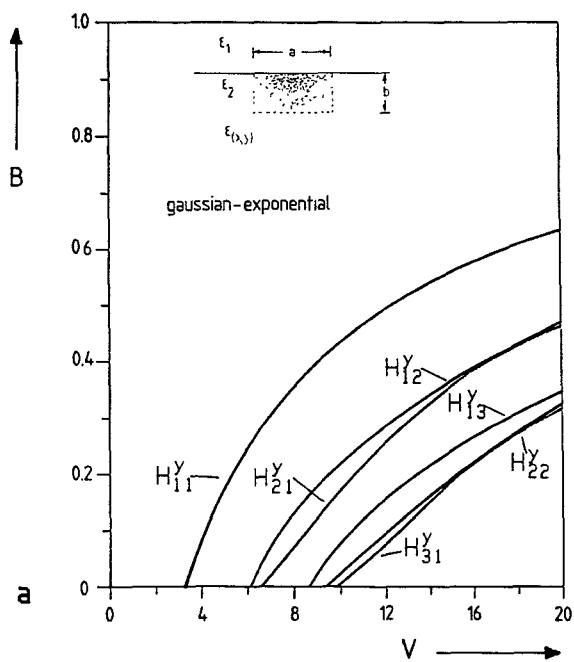


Fig. 5: Normalized propagation characteristic  $B$  versus normalized frequency  $V$ . a) Diffused channel waveguide with a Gaussian-exponential index profile.  $a/b = 1$ ,  $\epsilon_1 = \epsilon_0$ ,  $\epsilon_2 = 2\epsilon_0$ ,  $\epsilon_{\max} = 2.25\epsilon_0$ .

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